ME 3304 Heat Transfer

Lecture Note (8) Mass Transfer

Prof. Changmin Son
changminson@vt.edu

Physical Origin

Transfer is due to random molecular motion.

Consider two species A and B at the same *T* and *p*, but initially separated by a partition.

- Diffusion in the direction of decreasing concentration dictates net transport of A molecules to the right and B molecules to the left.
- In time, uniform concentrations of A and B are achieved.

Mass diffusion occurs in liquids and solids, as well as in gases.

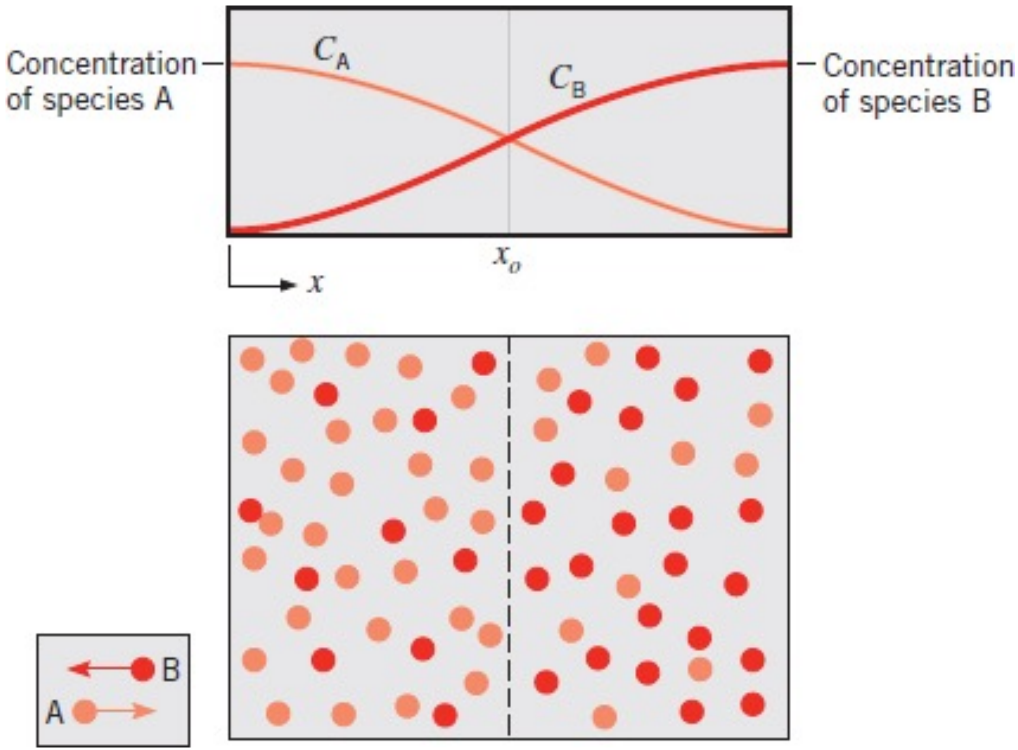


FIGURE 14.1 Mass transfer by diffusion in a binary gas mixture.

Mass Diffusion

The mass flow of a species per unit area (mass flux) is proportional to the concentration gradient. (Fick's law)

Mass Flux $n_A'' = \frac{\dot{m}_A}{A} = -D_{AB}\frac{\partial \rho_A}{\partial x}$ **Mole Flux** $N_A'' = -D_{AB}\frac{\partial C_A}{\partial x}$

$$n_A'' = N_A'' M_A$$

where,

$\dot{m}_A$: mass flow per unit time, [kg/s]
A: area, [m]
D: diffusion coefficient, [m^2/s]
ρ_A: density of species A, [kg/m^3]
C_A: molar concentration of species A per unit volume, [kmol/m^3]
M_A: molecular weight of species A, [kg/kmol]

The diffusivities for energy, species and momentum all have the same unit [m^2/s]

Other Diffusion

Transport of Energy
[Heat Conduction Equation]

$$\left(\frac{q}{A}\right)_x = -k\frac{\partial T}{\partial x}$$

Transport of Momentum across boundary layer
[Viscous-Shear Equation]

$$\tau = \mu\frac{\partial u}{\partial y}$$

Fourier's law

$$q_x = -k\frac{\partial T}{\partial x} = -\frac{k}{\rho C_p}\frac{\partial}{\partial x}(\rho C_p T) = -\alpha\frac{\partial}{\partial x}(\rho C_p T)$$

where, $\alpha = \frac{k}{\rho C_p}\,[m^2/s]$: thermal diffusivity

$$\tau = \mu\frac{\partial u}{\partial y} = \frac{\mu}{\rho}\frac{\partial}{\partial y}(\rho u) = \nu\frac{\partial}{\partial y}(\rho u)$$

where, $\upsilon = \frac{\mu}{\rho}\,[m^2/s]$: kinematic viscosity

Mass Transfer

Heat is transferred if there is a temperature difference in a medium.

Similarly, if there is a difference in the concentration of some chemical species in a mixture, mass transfer must occur.

"Mass transfer is mass in transit as the result of a species concentration difference in a mixture."

Just as a temperature gradient constitutes the driving potential for heat transfer, a species concentration gradient in a mixture provides the driving potential for transport of that species.

Mass and Molecular Concentration

Mass concentration

$$m_A = \frac{\rho_A}{\sum_{i=1}^{n} \rho_i} = \frac{\rho_A}{\rho}$$

Molecular (molar) concentration

$$C_A = \frac{\rho_A}{M_A}$$

where, M_A: molecular weight of species A

For gas phase

$$C_A = \frac{n_A}{V} = \left(\frac{m/M_A}{V} = \frac{\rho_A}{M_A}\right) = \frac{p_A}{RT}$$

where, $p_A V = n_A RT$

m: total mass of the gas

R: universal gas constant

Mole fraction of mixtures

$$x_A = \frac{C_A}{\sum_{i=1}^{n} C_i} = \frac{C_A}{C}$$

where, $C = n_{total}/V = P/RT$

Fick's Law - Diffusive Mass Flux

The rate equation for mass diffusion is known as **Fick's law**, and for the transfer of species A in a *binary mixture* of A and B, it may be expressed in vector form as

$$\boldsymbol{j}_A = -\rho D_{AB} \nabla m_A \qquad \text{or} \qquad \boldsymbol{J}_A^* = -C D_{AB} \nabla x_A$$

The quantity $\boldsymbol{j}_A$ (kg/s · m²) is defined as the diffusive **mass flux** of species *A*.

It is the amount of *A* that is transferred by diffusion per unit time and per unit area perpendicular to the direction of transfer, and it is proportional to the mixture mass density, $\rho = \rho_A + \rho_B$ (kg/m³), and to the gradient in the species mass fraction (**mass concentration**), $\boldsymbol{m_A} = \rho_A/\rho$.

Fick's law defines a second important transport property, namely, the *binary diffusion coefficient* or *mass diffusivity (diffusion coefficient)*, D_{AB}.

The species flux may also be evaluated on a molar basis, where J^*_A (kmol/s·m²) is the diffusive **molar flux** of species A. It is proportional to the **total molar concentration** of the mixture, $C = C_A + C_B$ (kmol/m³), and to the gradient in the species **mole fraction**, $\boldsymbol{x_A} = C_A/C$.

Physical Origin - Mass Diffusivity

Considerable attention has been given to predicting the mass diffusivity D_{AB} for the binary mixture of two gases, A and B.

Assuming ideal gas behavior, kinetic theory may be used to show that

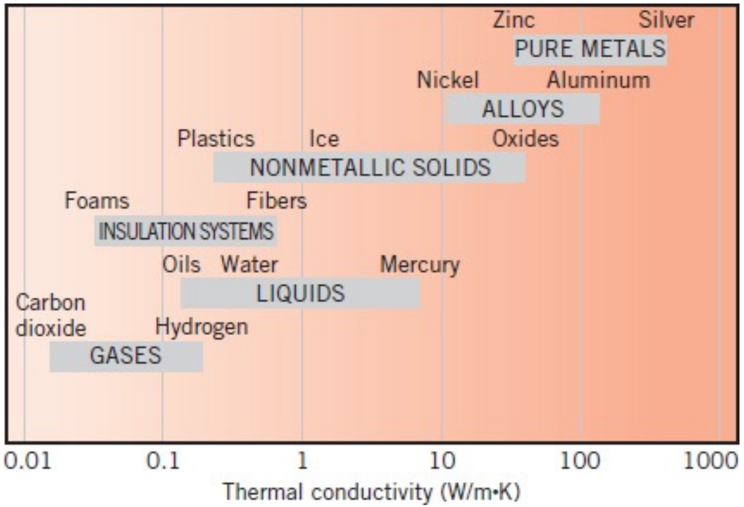


FIGURE 2.4 Range of thermal conductivity for various states of matter at normal temperatures and pressure.

$$D_{AB} \approx (k) = \frac{1}{3}\bar{c}\lambda_{mfp} \sim p^{-1}T^{3/2}$$

where T is expressed in kelvins

$\bar{c}$: electron mean velocity

λ_{mfp}: electron mean free path

Physical Origin - Mixture Composition

The mass of species *i* per unit volume of the mixture $$\rho_i = \mathscr{M}_i C_i$$

Mass fraction (the amount of species *i* in a mixture) $$m_i = \frac{\rho_i}{\rho} \Longrightarrow \sum_i m_i = 1$$

Mole fraction $$x_i = \frac{C_i}{C} = \frac{p_i}{p} \Longrightarrow \sum_i x_i = 1$$

Ideal gas law for a mixture $$\rho_i = \frac{p_i}{R_i T}$$

Molar concentration $$C_i = \frac{\rho_i}{M_i} = \frac{p_i}{RT}$$

where, R_i is the gas constant for species i, and R is the universal gas constant

Problem 14.10

An old-fashioned glass apothecary jar contains a patent medicine. The neck is closed with a rubber stopper that is 20 mm tall, with a diameter of 15 mm at the bottom end, widening to 20 mm at the top end. The molar concentration of medicine vapor in the stopper is 2×10^{-3} kmol/m^3 at the bottom surface and is negligible at the top surface. If the mass diffusivity of medicine vapor in rubber is 0.15×10^{-9} m^2/s, find the rate (kmol/s) at which vapor exits through the stopper.

Nomenclature

C_i : Molar concentration $\left(\text{kmol/m}^3\right)$ of species *i*.

ρ_i : Mass density (kg/m^3) of species *i*.

$\mathcal{M}_i$: Molecular weight (kg/kmol) of species *i*.

$$\rho_i = \mathcal{M}_i C_i$$

J_i^* : Molar flux $\left(\text{kmol/s}\cdot\text{m}^2\right)$ of species *i* due to diffusion.

- ➢ Transport of *i* relative to molar average velocity (υ*) of mixture.

N_i'': Absolute molar flux $\left(\text{kmol/s}\cdot\text{m}^2\right)$ of species *i*.

- ➢ Transport of *i* relative to a fixed reference frame.

j_i : Mass flux $\left(\text{kg/s}\cdot\text{m}^2\right)$ of species *i* due to diffusion.

- ➢ Transport of *i* relative to mass-average velocity (υ) of mixture.

n_i'': Absolute mass flux $\left(\text{kg/s}\cdot\text{m}^2\right)$ of species *i*.

- ➢ Transport of *i* relative to a fixed reference frame.

x_i : Mole fraction of species *i* $\left(x_i = C_i / C\right)$.

m_i : Mass fraction of species *i*$\left(m_i = \rho_i / \rho\right)$.

Mass and Molecular Velocities

In a mixture, the various species will normally move at different velocities.

Mass average velocity for a mixture

$$\mathbf{v} = \frac{\sum_{i=1}^{n} \rho_i \mathbf{v}_i}{\sum_{i=1}^{n} \rho_i} = \frac{\sum_{i=1}^{n} \rho_i \mathbf{v}_i}{\rho}$$

Molar average velocity for a mixture

$$\mathbf{V} = \frac{\sum_{i=1}^{n} C_i \mathbf{v}_i}{\sum_{i=1}^{n} C_i} = \frac{\sum_{i=1}^{n} C_i \mathbf{v}_i}{C}$$

Diffusion velocity

$\mathbf{v}_i - \mathbf{v}$ The diffusion velocity of species $\boldsymbol{i}$ relative to the mass average velocity

$\mathbf{v}_i - \mathbf{V}$ The diffusion velocity of species $\boldsymbol{i}$ relative to the molar average velocity

Mass Transfer in Nonstationary Media (bulk motion)

If there is **bulk motion**, then, like heat transfer, mass transfer can also occur by **advection**.

However, the **diffusion** of a species always involves the movement of molecules or atoms from one location to another. In many cases, this molecular scale motion results in bulk motion.

We define the **total** or *absolute* ***flux*** of a species, which includes both **diffusive and advective components.**

Total (absolute) flux = advection + diffusion

$$\boldsymbol{n}_i'' = M_i \boldsymbol{N}_i''$$

$$\mathbf{v}_i = \frac{\boldsymbol{N}_i''}{C_i} = \frac{\boldsymbol{n}_i''}{\rho_i}$$

$$\boldsymbol{n}_i'' = \boldsymbol{j}_i + \rho_i \mathbf{v} \qquad \boldsymbol{N}_i'' = \boldsymbol{J}_i^* + C_i \mathbf{v}$$

Absolute and Diffusive Species Fluxes

$$\boldsymbol{j}_A = -\rho D_{AB} \nabla m_A$$

$$\boldsymbol{n}'' = \left(\boldsymbol{n}_A'' + \boldsymbol{n}_B''\right) = \rho \mathbf{v} = \rho_A \mathbf{v}_A + \rho_B \mathbf{v}_B$$

$$\boldsymbol{n}_A'' = \boldsymbol{j}_A + \rho_A \mathbf{v}$$

$$\boldsymbol{n}_A'' = -\rho D_{AB} \nabla m_A + m_A\left(\boldsymbol{n}_A'' + \boldsymbol{n}_B''\right)$$

$$\boldsymbol{J}_A^* = -C D_{AB} \nabla x_A$$

$$\boldsymbol{N}'' = \left(\boldsymbol{N}_A'' + \boldsymbol{N}_B''\right) = C \mathbf{v} = C_A \mathbf{v}_A + C_B \mathbf{v}_B$$

$$\boldsymbol{N}_A'' = \boldsymbol{J}_A^* + C_A \mathbf{v}$$

$$\boldsymbol{N}_A'' = -C D_{AB} \nabla x_A + x_A\left(\boldsymbol{N}_A'' + \boldsymbol{N}_B''\right)$$

$\boldsymbol{n}_i''$: Mass flux of species *i* relative to a set of fixed coordinates

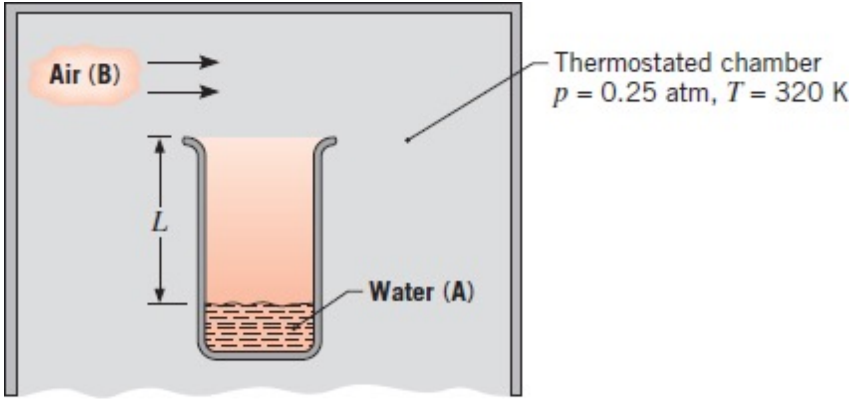
$\boldsymbol{N}_i''$: Molar flux of species *i* relative to a set of fixed coordinates

M_i: Molecular weight of species *i* relative to a set of fixed coordinates

v: Velocity of species *i* relative to a set of fixed coordinates

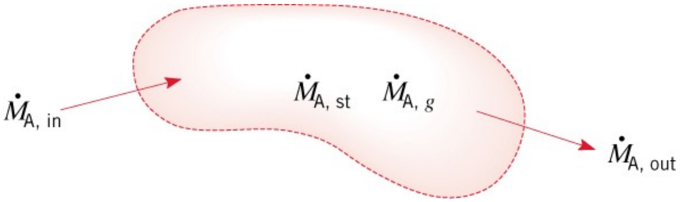
Problem 14.15

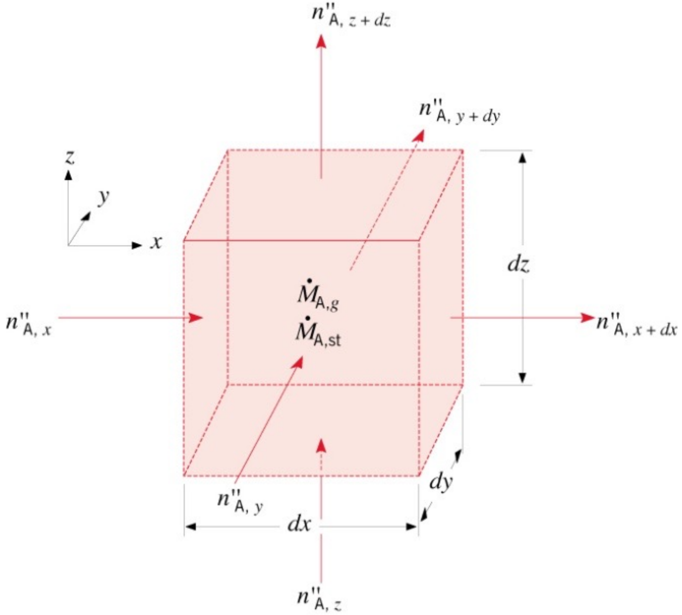
A laboratory apparatus to measure the diffusion coefficient of vapor–gas mixtures consists of a vertical, small-diameter column containing the liquid phase that evaporates into the gas flowing over the mouth of the column. The gas flow rate is sufficient to maintain a negligible vapor concentration at the exit plane. The column is 200 mm high, and the pressure and temperature in the chamber are maintained at 0.25 atm and 320 K, respectively. Calculate the expected evaporation rate ($kg/h \cdot m^2$) for a test with water and air under the foregoing conditions, using the known value of D_{AB} for the vapor–air mixture.



14.4 Conservation of Species for a Stationary Medium

$$\dot{M}_{A,in} - \dot{M}_{A,out} + \dot{M}_{A,g} = \frac{dM_A}{dt} \equiv \dot{M}_{A,st}$$





If D_{AB}, C and ρ are constant:

Species Diffusion Equation on a Molar Basis:

$$\frac{\partial^2 C_A}{\partial x^2} + \frac{\partial^2 C_A}{\partial y^2} + \frac{\partial^2 C_A}{\partial z^2} + \frac{\dot{N}_A}{D_{AB}} = \frac{1}{D_{AB}} \frac{\partial C_A}{\partial t}$$

Species Diffusion Equation on a Mass Basis:

$$\frac{\partial^2 \rho_A}{\partial x^2} + \frac{\partial^2 \rho_A}{\partial y^2} + \frac{\partial^2 \rho_A}{\partial z^2} + \frac{\dot{n}_A}{D_{AB}} = \frac{1}{D_{AB}} \frac{\partial \rho_A}{\partial t}$$

Ratio of Different Transport Processes

Prandtl number

$$\mathrm{Pr} = \frac{\text{viscous momentum transfer}}{\text{heat conduction}} = \frac{\upsilon}{\alpha} = \left(\frac{\mu}{\rho} \cdot \frac{\rho C_p}{k}\right) = \frac{\mu C_p}{k}$$

Schmidt number

$$\mathrm{Sc} = \frac{\text{viscous momentum transfer}}{\text{species diffusion}} = \frac{\upsilon}{D_{AB}} = \frac{\mu}{\rho D_{AB}}$$

Lewis number

$$\mathrm{Le} = \frac{\alpha}{D_{AB}} = \frac{k}{\rho C_p D_{AB}} = \frac{\mathrm{Sc}}{\mathrm{Pr}}$$

Mass Transfer Coefficient (Chapter 6)

$$\dot{m}_A = h_m A\left(C_{A,1} - C_{A,2}\right)$$

where,
$\dot{m}_A$: diffisive mass flow of component A, $[kg/s]$
h_m: mass transfer coefficient, [m/s]
$C_{A,1}, C_{A,2}$: concentration, [kg/m^3]

Sherwood number $Sh = \dfrac{h_m x}{D_{AB}} = f(Rex, Sc) \Longleftrightarrow Nu = \dfrac{hx}{k} = f(Rex, Pr)$

Empirical correlation by Gilliland (1934) $Sh = \frac{h_m d}{D} = 0.023 Re^{0.83} \times Sc^{0.44}$

Problem 6.42

On a summer day the air temperature is 30°C and the relative humidity is 55%. Water evaporates from the surface of a lake at a rate of 0.08 kg/h per square meter of water surface area. The temperature of the water is also 30°C. Determine the value of the convection mass transfer coefficient.